Title: Solving Quadratic Equations by Factoring
Class: Math 100 or Math 107
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Instructions to tutor: Read instructions and follow all steps for each problem exactly as given.
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Solving Quadratic Equations by Factoring

Purpose:
This is intended to refresh your knowledge about solving quadratic equations using factoring methods.

Recall that a **quadratic equation** is an equation that can be written in the form $ax^2 + bx + c = 0$, with $a \neq 0$. For example, $3x^2 + 4x - 7 = 0$, $6 - x^2 = 2x$, and $x(x+6) = 14$ are all quadratic equations. Note that the second two equations would require a couple algebraic steps to be put into the form shown above.

Once we have written a quadratic equation in the form $ax^2 + bx + c = 0$, we will use the following property to help us solve the equation. The property will allow us to break the quadratic equation into two simpler linear equations.

Zero-Product Property
Suppose that A and B are two algebraic expressions. Then if $A \cdot B = 0$, either $A = 0$ or $B = 0$.

Example: Solve $x^2 - 5x - 6 = 0$.

First note that this is a quadratic equation in the form $ax^2 + bx + c = 0$.

Our next step is to factor – note that $x^2 - 5x - 6 = (x-6)(x+1)$.

So $x^2 - 5x - 6 = 0 \quad \Rightarrow \quad (x-6)(x+1) = 0$. What do you notice about our new equation?

The equation $(x-6)(x+1) = 0$ is in the form $A \cdot B = 0$. If the product of two terms is zero then at least one of the individual terms is zero.

Applying the zero-product property, $(x-6)(x+1) = 0 \quad \Rightarrow \quad x - 6 = 0$ or $x + 1 = 0$.

Note that the two new equations are linear, and we already know how to solve linear equations.

So $x - 6 = 0$ or $x + 1 = 0$ leads us to $x = 6$ or $x = -1$, our two solutions. You should substitute these into the original equation to see that they check out.

Without the narrative:

$$
\begin{aligned}
& x^2 - 5x - 6 = 0 \\
\Rightarrow \quad & (x-6)(x+1) = 0 \\
\Rightarrow \quad & x - 6 = 0 \ \text{ or } \ x + 1 = 0
\end{aligned}
$$

$$\Rightarrow \quad x = 6 \quad \text{or} \quad x = -1$$

Example: Now it's your turn. Solve $3x^2 + 10x - 8 = 0$.

Factor $(\_\_\_\_\_\_\_\_\_)(\_\_\_\_\_\_\_\_\_) = 0$

Zero-product $\_\_\_\_\_\_\_\_\_\_ = 0$ or $\_\_\_\_\_\_\_\_\_\_ = 0$

Solve $x = \_\_\_\_\_\_\_\_\_$ or $x = \_\_\_\_\_\_\_\_\_$

Did you get the solutions $x = \frac{2}{3}, -4$? Good! Let's try one which requires a little more work.

Example: Solve $x(x+2) = 8$.

First, what is wrong with starting this as $x = 8$ or $x + 2 = 8$? Correct, we do not have the product of two terms set equal to zero (there are lots of ways to make a product of 8!). So we must first write the quadratic in the form $ax^2 + bx + c = 0$.

So $x(x+2) = 8 \quad \Rightarrow \quad x^2 + 2x = 8 \quad \Rightarrow \quad x^2 + 2x - 8 = 0$.

Now you can finish this problem like the previous example.

Factor $(\_\_\_\_\_\_\_\_\_)(\_\_\_\_\_\_\_\_\_) = 0$

Zero-product $\_\_\_\_\_\_\_\_\_\_ = 0$ or $\_\_\_\_\_\_\_\_\_\_ = 0$

Solve $x = \_\_\_\_\_\_\_\_\_$ or $x = \_\_\_\_\_\_\_\_\_$

Did you get the solutions $x = 2, -4$? Good! Now you are on your own for the next few problems.

1. Solve each of the following quadratic equations.

   (a) $2x(x-8) = 0$

   (b) $2x^2 + 7x + 3 = 0$

2. Solve each of the following quadratic equations.

(a) $y^2 + 20 = 9y$

(b) $a(2a - 13) = -20$

Check your answers – If you did not get these, consult a tutor for help.

1. (a) $x = 0, 8$ (b) $x = -\frac{1}{2}, -3$

2. (a) $y = 4, 5$ (b) $a = \frac{2}{3}, -4$

If the previous problems went smoothly, then you are ready to try a couple problems that are a little more advanced. Neither of these is quadratic, but can still be solved by factoring and using the zero-product property. Try other factoring methods you have learned in class, such as factoring out greatest common factors and factoring by grouping.

3. Solve each of the following by factoring.

(a) $3u^3 - 5u^2 - 2u = 0$

(b) $x^4 - 5x^3 - 9x^2 + 45x = 0$

Check your answers – If you did not get these, consult a tutor for help.

3. (a) $u = -\frac{1}{3}, 0, 2$ (b) $x = -3, 0, 3, 5$